Name:______________________________ Date:________________________

Math 10 Honours: Ch2 Contest Questions on Sequences and Series

1. The geometric sequence with "n" terms $t_1, t_2, t_3, ...t_{n-1}, t_n$ has $t_1 t_n = 3$. Also, the product of all "n" terms equals 59049. That is $\left(t_1 \times t_2 \times ...t_{n-1} \times t_n = 59049\right)$ Determine the value of "n" [Euclid 2014]
   a. Write a formula in terms of $a^m \times r^p = 59049$

   b. Show your steps on how to find "n"

2. In the Fibonacci sequence, 1, 1, 2, 3, 5, …… each term after the second is the sum of the previous two terms. How many of the first 100 terms of the Fibonacci sequence are odd?

3. In an arithmetic sequence, the sum of the first and third terms is 6 and the sum of the second and fourth terms is 20. Determine the tenth term in the sequence.

4. Jackson gave the following rule to create a sequence: [Galois]

"If x is a term in your sequence, then the next term in your sequence is $\frac{1}{1-x}$."
For example, Mary starts her sequence with the number 3.
The second term of her sequence is $\frac{1}{1-3} = \frac{1}{-2} = -\frac{1}{2}$. Her sequence is now $3, -\frac{1}{2}$.

The third term of her sequence is $\frac{1}{1-(-\frac{1}{2})} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$. Her sequence is now $3, -\frac{1}{2}, \frac{2}{3}$.

The fourth term of her sequence is $\frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3$. Her sequence is now $3, -\frac{1}{2}, \frac{2}{3}, 3$.

Fabien starts his sequence with the number 2, and continues using Jackson's rule until the sequence has 2011 terms.

a) What is the second term of his sequence?

b) What is the fifth term of his sequence?

c) How many of the 2011 terms in Fabien's sequence are equal to 2? Explain

d) Determine the sum of all of the terms in his sequence

5. A) Find the 11th term in the arithmetic sequence: 17, 22, 27, 32…. [Galois]

b) Explain why there is no number which occurs in both of the following arithmetic sequences:
sequence 1: 17, 22, 27, 32,….. Sequence 2: 13, 28, 43, 58……..

c) Find all the numbers less than 420 which occurs in both of the following arithmetic sequences:

Sequence 1: 17, 22, 27, 32,…. Sequence 2: 16, 22, 28, 34, ……

6. A) Three terms of an arithmetic sequence adds to 180. Determine the middle term [Cemc]

   b) Five terms of an arithmetic sequence adds to 180. Show that atleast one of the five terms equals 36

   c) Six terms of an arithmetic sequence adds to 180. Determine the sum of the first and sixth terms of the sequence.

7. The peizi-sum of a sequence $a_1, a_2, a_3, ...a_n$ is formed by adding the products of all of the pairs of distinct terms in the sequence. For example, the peizi-sum of the sequence a_1, a_2, a_3, a_4 is
   $P_z Sum = a_1 \times a_2 + a_1 \times a_3 + a_1 \times a_4 + a_2 \times a_3 + a_2 \times a_4 + a_3 \times a_4$ [Hypatia]

   a) The peizi-sum of the sequence $2, 3, x, 2x$ is equal to -7. Determine the value of "x"

   b) A sequence has 100 terms. Of these terms, "m" are equal to 1 and "n" are equal to -1. The rest of the terms are equal to 2. Determine in terms of "m" and "n", the number of pairs of distince terms that have a product of 1

   c) A sequence has 100 terms, with each term equal to either 2 or -1. Determine, with justification, the minimum possible peizi-sum of the sequence:

8. Find the sum of "N": $N = 100^2 + 99^2 - 98^2 - 97^2 + 96^2 + ... + 4^2 + 3^2 - 2^2 - 1^2$. The addition and subtractions of terms occur alternately in pairs. [aime 2008]

9. Determine all right triangles where all sides form an arithmetic sequence, where none of the sides are equal, and one of the sides must be equal to 60. Find the dimensions of all possible triangles.

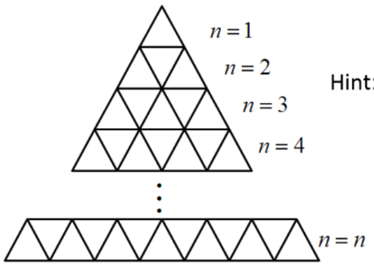
10. There are two sequences

$$a_0 = 1 \ \ and \ \ a_1 = 1 \qquad a_n = a_{n-1} + \frac{(a_{n-1})^2}{a_{n-2}} \ \ for \ n \geq 2$$

$$b_0 = 1 \ \ and \ \ b_1 = 3 \qquad b_n = b_{n-1} + \frac{(b_{n-1})^2}{b_{n-2}} \ \ for \ n \geq 2$$

What is the value of $\frac{b_{32}}{a_{32}}$? [AIME 2008]

11. Given the triangular image, derive a formula for the number of *upright* triangles [*triangles pointing up*] for "n" number of rows. For instance, for one row, there is 1 upright-triangle. For two rows, we have four upright-triangles, for three rows we have 10 upright triangles. How many upright triangles if we have "n" rows?



Hint: $1^2+2^2+3^2+....+n^2=\frac{n(n+1)(2n+1)}{6}$

12. For each positive integer "N", an Eden sequence from {12,3,4,…N} is defined to be a sequence that satisfies the following conditions: [Euclid]

(i) each of its terms is an element of the set of consecutive integers $\{1,2,3,\ldots,N\}$,

(ii) the sequence is increasing, and

(iii) the terms in odd numbered positions are odd and the terms in even numbered positions are even.

For example, the four Eden sequences from $\{1,2,3\}$ are

$$1 \qquad 3 \qquad 1,2 \qquad 1,2,3$$

a) Determine the number of Eden sequences from {1,2,3,4,5}

b) For each positive integer "N", define $e(N)$ to be the number of Eden sequences from $\{1,2,3,4,5...N\}$. If $e(17)=4180$ and $e(20)=17710$, determine $e(18)$ and $e(19)$.

Answers:

1. N=20 2) 67 Odd terms. 3) Easy 4) A) -1, b) -1, C) 671 terms equal to 2 D) 1007

5) A) 67 B) easy C) 22, 52, 82, 112, 142, 172, 202, 232, 262, 292, 322, 352, 382, and 412.

6) A) 60 B) 180/5=36 C) 72 7) A) x=-1, x=-13/2 b) $0.5m(m-1)+0.5n(n-1)$ C) -99

8) Sum = 10100 9) 60, 80, 100, & 45,60,75, & 36, 48, 60 10) b32/a32=561 11) C $\frac{n(n+1)(2n+1)}{12}+\frac{n(n+1)}{4}$

12) a) there are 12 such sequences: $1 \quad 3 \quad 5 \quad 1,2 \quad 1,4 \quad 3,4 \quad 1,2,3 \quad 1,2,5 \quad 1,4,5 \quad 3,4,5 \quad 1,2,3,4 \quad 1,2,3,4,5$

b) $e(18)=6764$ and $e(19)=6764+4181=10945.$